

Example: $r=3$ Receive antennas
 $t=2$ Transmit antennas.

3 X 2 MIMO system

Transmit vector = $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

Transmitted on 1st antenna

Transmitted on 2nd antenna.

2 Dimensional Transmit vector

$$\text{Receive vector} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

3 Dimensional
Receive symbol vector

$$H = \begin{matrix} 3 \times 2 \\ r \times t \end{matrix} \text{ matrix} \quad \begin{matrix} 6 \text{ coefficients} \end{matrix}$$

$$\underbrace{\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}}_{3 \text{ Dim } \bar{y}} = \underbrace{\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \\ h_{31} & h_{32} \end{bmatrix}}_{3 \times 2 \text{ matrix } H} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\substack{2 \text{ Dim} \\ \bar{x}}} + \underbrace{\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}}_{3 \text{ Dim } \bar{w}}$$

$$\begin{aligned}
 y_1 &= h_{11} x_1 + h_{12} x_2 + w_1 \\
 y_2 &= h_{21} x_1 + h_{22} x_2 + w_2 \\
 y_3 &= h_{31} x_1 + h_{32} x_2 + w_3
 \end{aligned}$$

In general rxt system

$$y_l = h_{l1}x_1 + h_{l2}x_2 + \dots + h_{lt}x_t + w_l$$

$\uparrow$
symbol on
 l^{th} receive
antenna

Special Cases:

SIMO

$$t = 1$$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_r \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_r \end{bmatrix} x + \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_r \end{bmatrix}$$

Multiple Receive antenna.

SIMO - Single Input Multiple output system

$$r = 1$$

number of Rx antennas = 1

$$y_1 = \underbrace{[h_1 \ h_2 \ \dots \ h_t]}_{\text{channel vector}} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_t \end{bmatrix} + w_1$$

MISO
Multiple Input
Single Output

$$y_1 = \bar{h}^H \bar{x} + w_1$$

$$\bar{h} = \begin{bmatrix} h_1^* \\ h_2^* \\ \vdots \\ h_t^* \end{bmatrix} \quad \bar{h}^H = [h_1 \ h_2 \ \dots \ h_t]$$

$$r=1, t=1$$

$$y = h x + w$$

Fading channel
coefficient

Termed as SISO
Single Input Single
Output wireless
system.

$$\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_t \end{bmatrix}$$

r Dimensional
noise vector

Each w_i is Gaussian

mean 0

variance σ^2

$$E\{w_i\} = 0$$

$$E\{|w_i|^2\} = E\{w_i w_i^*\} = \sigma^2$$

Further, W_i are iid

Independent Identically
Distributed.

$$\begin{aligned} E\{W_i W_j^*\} &= E\{W_i\} E\{W_j^*\} \\ &= 0 \text{ if } i \neq j \end{aligned}$$

$$E\{W_i W_j^*\} = \begin{cases} \sigma^2 & \text{if } i=j \\ 0 & \text{if } i \neq j \end{cases}$$

$$E\{\bar{W} \bar{W}^H\} = \text{Covariance matrix}$$

$$= E\left\{ \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_r \end{bmatrix} \begin{bmatrix} W_1^* & W_2^* & \dots & W_r^* \end{bmatrix} \right\}$$

$$= E \left\{ \begin{matrix} \sigma^2 & 0 & 0 \\ W_1 W_2^* & W_1 W_3^* & \dots \\ W_2 W_1^* & |W_2|^2 & \\ W_3 W_1^* & & \\ \vdots & & \\ & & |W_r|^2 \end{matrix} \right\}$$

$$= \begin{bmatrix} \sigma^2 & 0 & \dots & 0 \\ 0 & \sigma^2 & & \\ \vdots & & \ddots & \\ 0 & & & \sigma^2 \end{bmatrix}$$

$$= \sigma^2 \mathbf{I}_{r \times r \text{ Dimensional Identity matrix}}$$

Example: Consider a MIMO system with $r = 3$.

$$\sigma^2(\text{dB}) = -3 \text{ dB}$$

Considering IID noise elements,
Noise covariance = ?

$$R = \sigma^2 I_{3 \times 3}$$

$$\sigma^2(\text{dB}) = -3 \text{ dB}$$

$$10 \log_{10} \sigma^2 = -3$$

noise
variance
matrix.

$$\Rightarrow \sigma^2 = 10^{-0.3} \approx \frac{1}{2}$$

$$R = \frac{1}{2} I_{3 \times 3} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$$